



MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

Model Order Reduction of Energy Networks with a Focus on Hyperbolic Systems

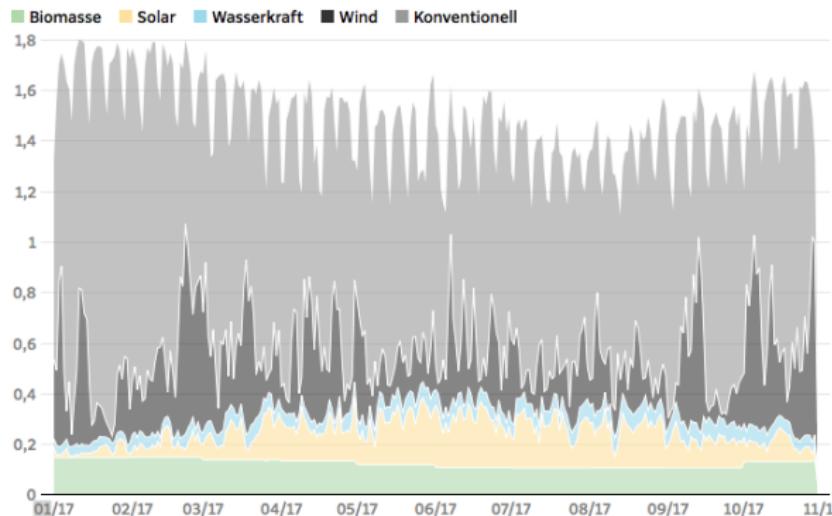
ICERM Virtual Workshop March 23rd -27th

Sara Grundel

March 24, 2020



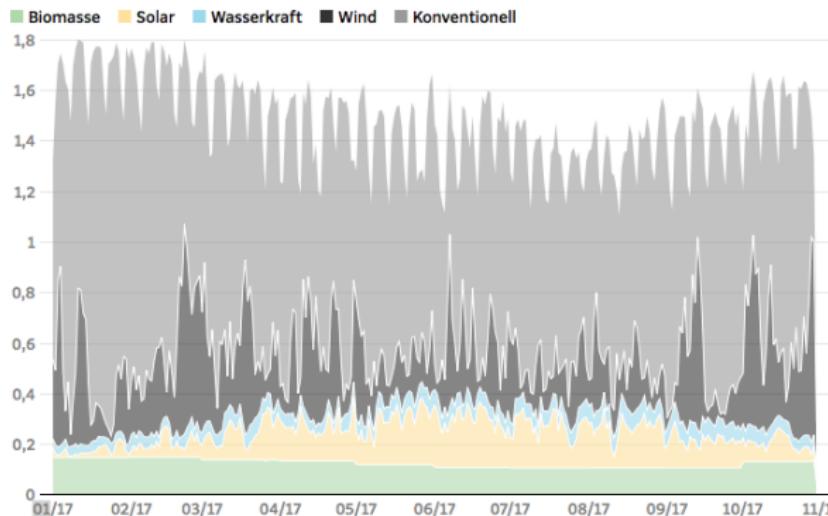
Future





How could 2050 look like?

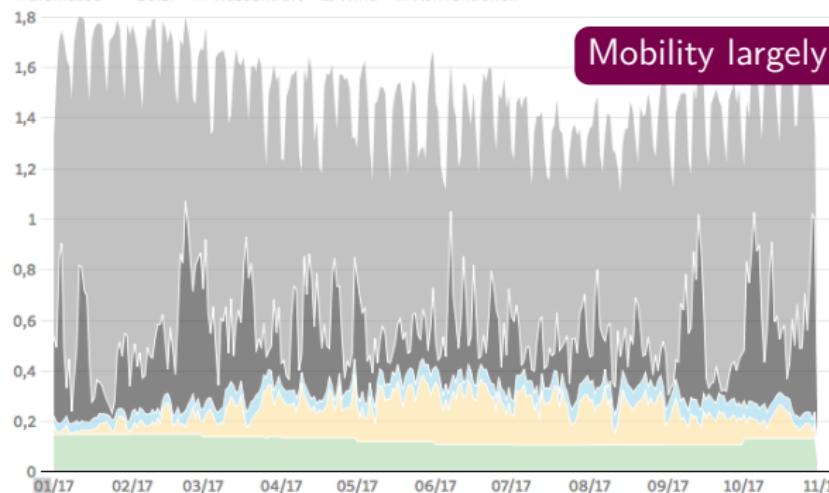
Renewable Energies



Quelle: 50 Hertz, Amprion, Tennet, TransnetBW, Destatis, EEX • Rohdaten herunterladen



Biomasse Solar Wasserkraft Wind Konventionell



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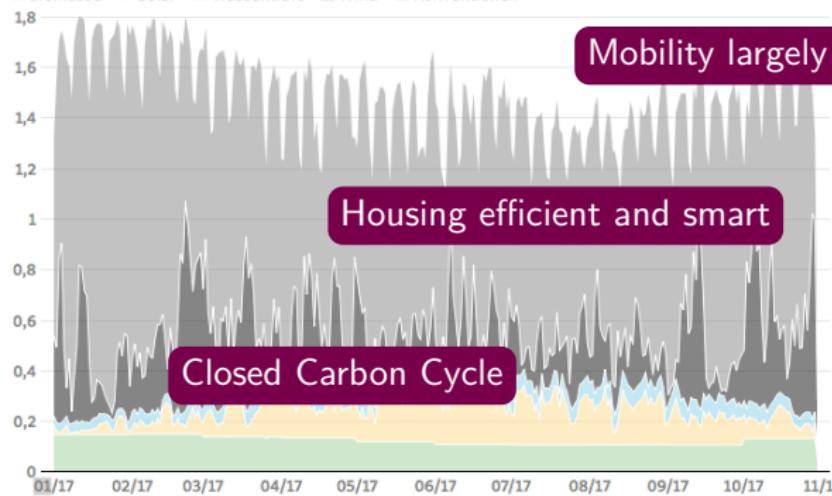
Mobility largely electric

Housing efficient and smart

Quelle: 50 Hertz, Amprion, Tennet, TransnetBW, Destatis, EEX • Rohdaten herunterladen



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Mobility largely electric

Housing efficient and smart

Closed Carbon Cycle

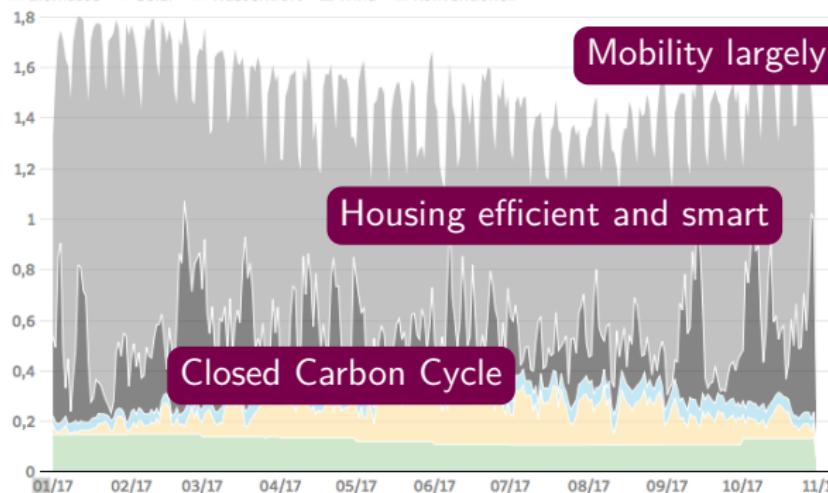
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Future

Renewable Energies

Biomasse Solar Wasserkraft Wind Konventionell





- Global Optimal Solutions of the entire energy system
- Each subsystem has its own simulation tool
- Efficient and fast simulation of each subsystem is wanted and probably needed! \Rightarrow Complexity and Dimension Reduction



Modeling Simulation Optimization

- Global Optimal Solutions of the entire energy system
- Each subsystem has its own simulation tool
- Efficient and fast simulation of each subsystem is wanted and probably needed! \Rightarrow Complexity and Dimension Reduction

Power Grid - different Levels

Smart Home - Control Centers

Gas transportation and storage networks

Energy conversion

Focus on gas distribution networks in this talk



Funding and Collaborators



EUROPÄISCHE UNION
Europäischer Fonds für
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and Energy



Federal Ministry
of Education
and Research

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Neeraj Sarna
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Philipp Sauerteig
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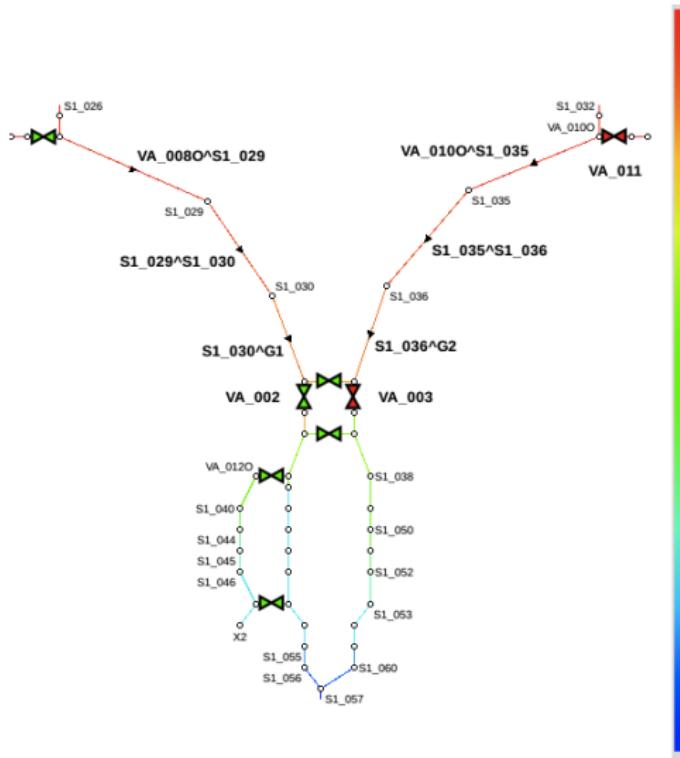


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1. Introduction/Motivation
2. Gas network model/PDAE
3. Discretization/Modeling of the isothermal Euler equation
4. Model Order Reduction based on ODEs
5. Feature Tracking Reduced Order Modelling for hyperbolic systems
6. Other Examples of Complexity Reduction in the context of the energy system



Gas transportation network



PDAE

- hyperbolic PDE on the pipe
- ODEs or algebraic equations on other components
- algebraic node conditions



Gas Transport in the Pipe

Isothermal Euler Equation

Density

- Transient
- Continuity

Gravity

- Linear
- Elevation

Friction

- Nonlinear
- Configurable

$$\begin{aligned}\frac{\partial \rho}{\partial t} &= -S^{-1} \frac{\partial q}{\partial x} \\ \frac{\partial q}{\partial t} &= -S \frac{\partial p}{\partial x} - S g \rho \frac{\partial h}{\partial x} - \frac{f_g}{2DS} \frac{q|q|}{\rho}\end{aligned}$$

Mass-Flux

- Transient
- Momentum

Gas State

- Pressure
- Density

Compressibility

- Nonlinear
- Configurable



- Overall structure is a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$.
- At each node in \mathcal{N} algebraic conditions are prescribed.
- The edges are the pipes described by the Euler equations.
- The resulting system looks like

$$\mathcal{M}\partial_t\phi(x, t) = \mathcal{K}\phi(x, t) + f(\phi(x, t), u(t), t)$$

which discretized is

$$M\dot{x} = Kx + Bu + f(x, t)^1,$$

where $\phi(x, t)$ is a vector of pressure and flux values at and $x(t)$ at different spatial points

- Depending on the network, the algebraic conditions used and the discretization schemes the matrices M, K, B and the function f can vary.
- In $u(t)$ the input functions are collected.

¹Benner, G., Himpe, Huck,Streubel, Gas Network Benchmark Models, Springer, 2018



- existence of solutions
- index concepts
- space discretization
- solver for the discretized PDAE (time integration)
- model order reduction (nonlinear, DAE, uncertain and parameterized)
- parameter optimization
- uncertainty quantification
- optimal control/ optimization



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Isothermal Euler and Discretization

Basic equation

$$\begin{aligned}\frac{\partial p}{\partial t} &= -\frac{1}{\gamma z S} \frac{\partial q}{\partial x} \\ \frac{\partial q}{\partial t} &= -S \frac{\partial p}{\partial x} - \frac{f_g \gamma z}{2DS} \frac{q|q|}{p}\end{aligned}$$

Naive Approach

$$\begin{aligned}\frac{\partial p^*}{\partial t} &= -\frac{1}{\gamma z S} \frac{q_R - q_L}{\Delta x} \\ \frac{\partial q^*}{\partial t} &= -S \frac{p_R - p_L}{\Delta x} - \frac{f_g \gamma z}{2DS} \frac{q^* |q^*|}{p^*}\end{aligned}$$

Decoupled approach

$$\begin{aligned}w^\pm &= \frac{1}{2}(q \pm \sqrt{\gamma z S} p) \\ \partial_t w^\pm \pm \frac{1}{\sqrt{\gamma z}} \partial_x w^\pm &= \frac{1}{2} f(q, p)\end{aligned}$$



One Pipe - Speed and Accuracy

Simulation of a Pipe

Δh	1000	300	50	10
Midpoint Discretization	68.36932	68.36932	68.36932	68.36932
Left/Right Discretization	68.36541	68.36834	68.36912	68.36928
Decoupled Discretization	68.36932	68.36932	68.36932	68.36932
True Value	68.36932	68.36932	68.36932	68.36932

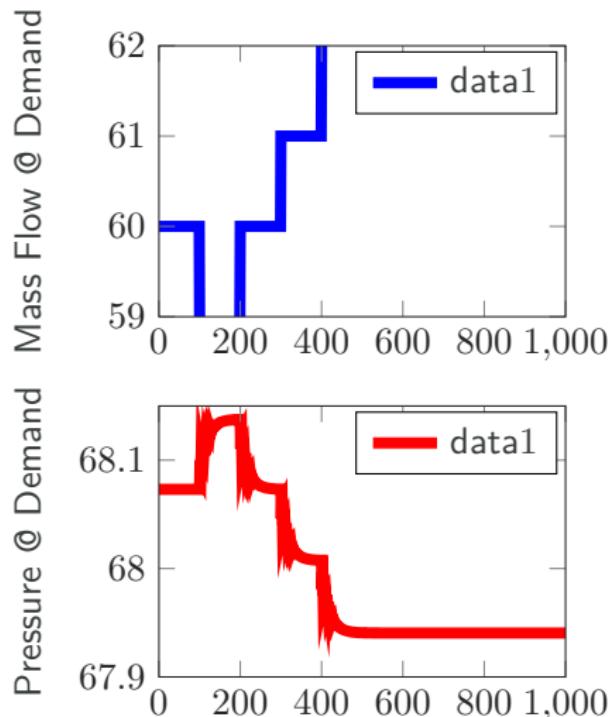
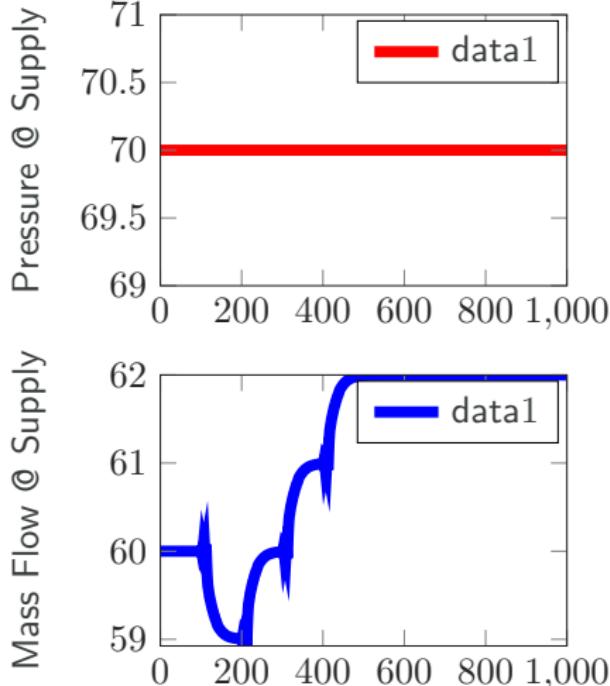
Table: Accuracy of the stationary solution

Δh	250	250	100	100	10
Solver	ode15s	IMEX	ode15s	IMEX	IMEX
Midpoint Discretization	4.97	0.02	35.9	0.03	0.18
LeftRight Discretization	1.29	0.01	2.67	0.02	0.11
Decoupled Discretization	1.22	0.01	1.93	0.02	0.09

Table: Speed of a simple simulation

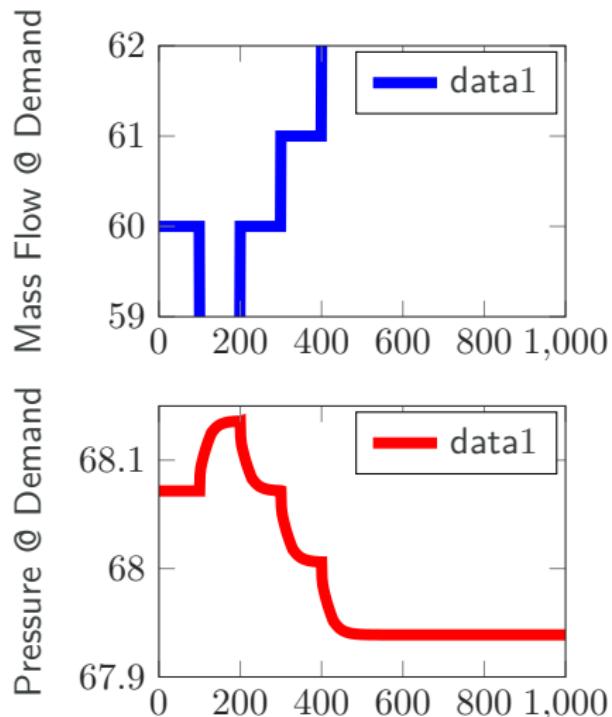
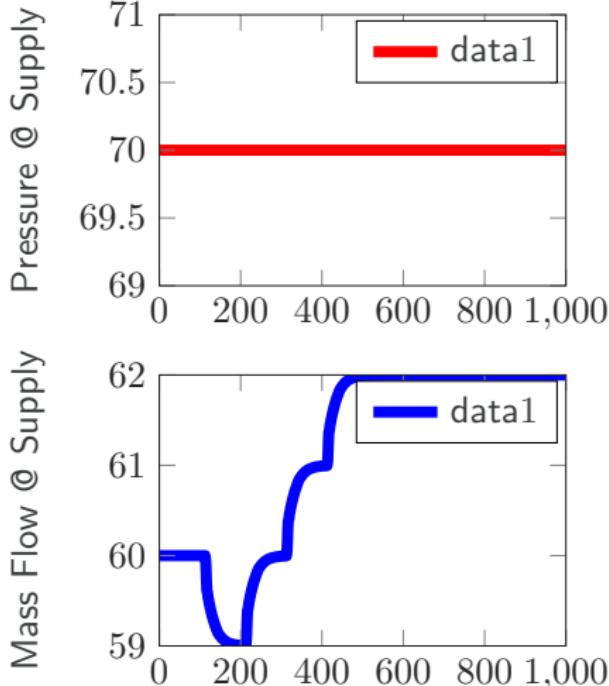


Dynamic Simulation Midpoint $\Delta h = 300$





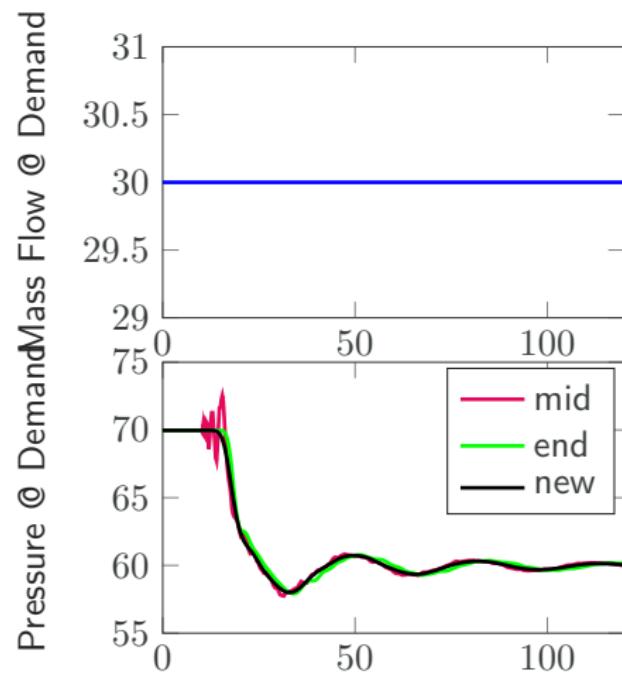
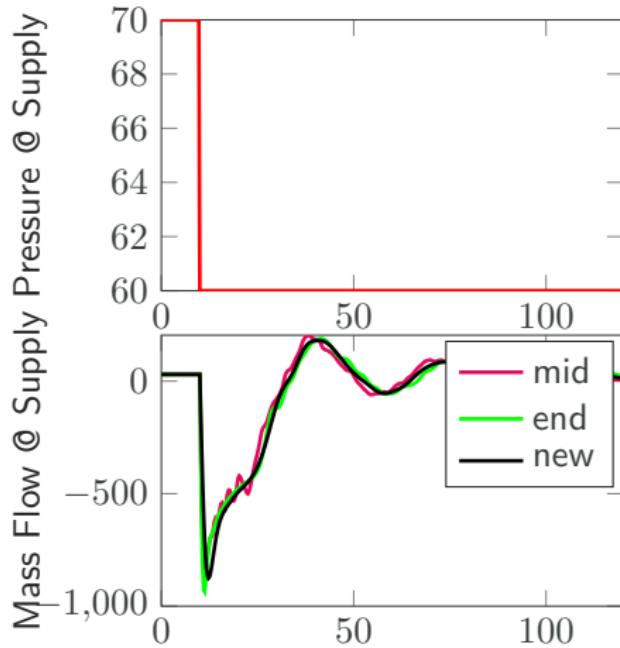
Dynamic Simulation Left/Right $\Delta h = 300$





Comparison

Numerical simulation of a pressure drop at the inlet of a pipe





Euler equation

$$\partial_t \rho(t, x) + \frac{1}{S} \partial_x q(t, x) = 0,$$

$$\frac{1}{S} \partial_t q(t, x) + \partial_x p(t, x) = -\frac{f_g}{2dS^2} \frac{q(t, x)|q(t, x)|}{\rho(t, x)}$$

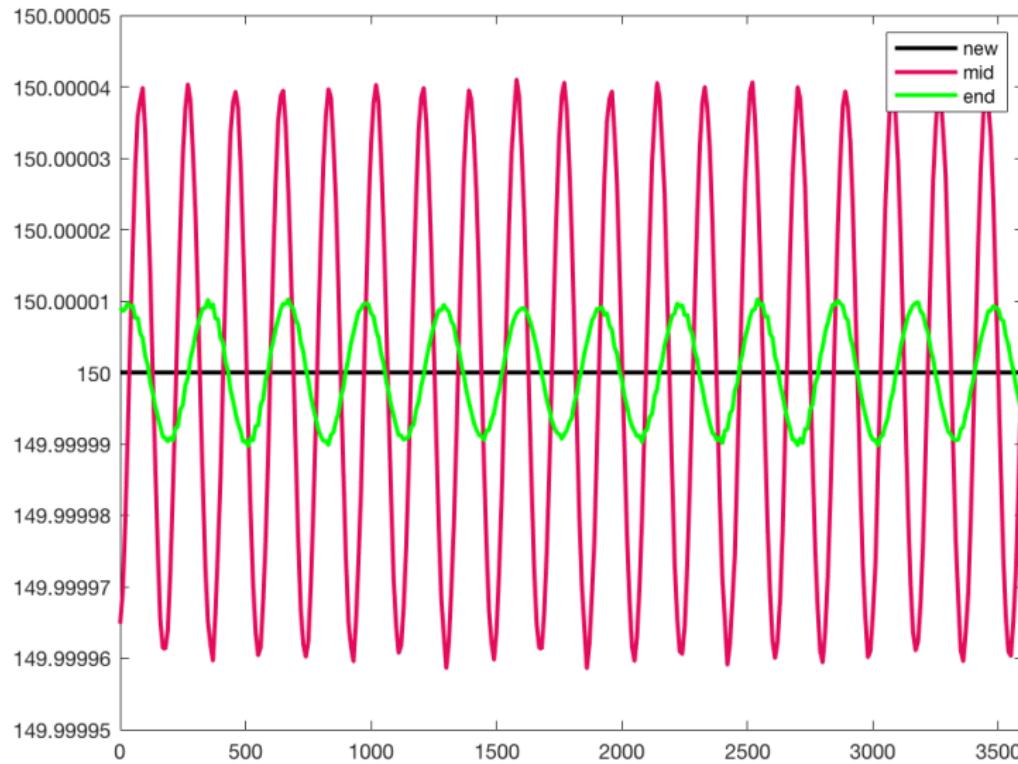
With $w^\pm(t, x) = \frac{1}{2} \left(\frac{1}{S} q + \int_0^\rho \lambda^\pm(s) ds \right)$ where $\lambda^\pm(\rho) = \pm \sqrt{\partial_\rho p(\rho)}$ we get

$$\partial_t w^\pm(t, x) + \lambda^\pm \partial_x w^\pm(t, x) = -\frac{1}{2} \frac{f_g}{2dS^2} (\rho u)(w^+, w^-)(t, x) |u(w^+, w^-)(t, x)|.$$

- S. Grundel, M. Herty, **Hyperbolic Discretization via Riemann Invariants** submitted



Oszillations of mass flux at the inlet in steady state





A network with cycles

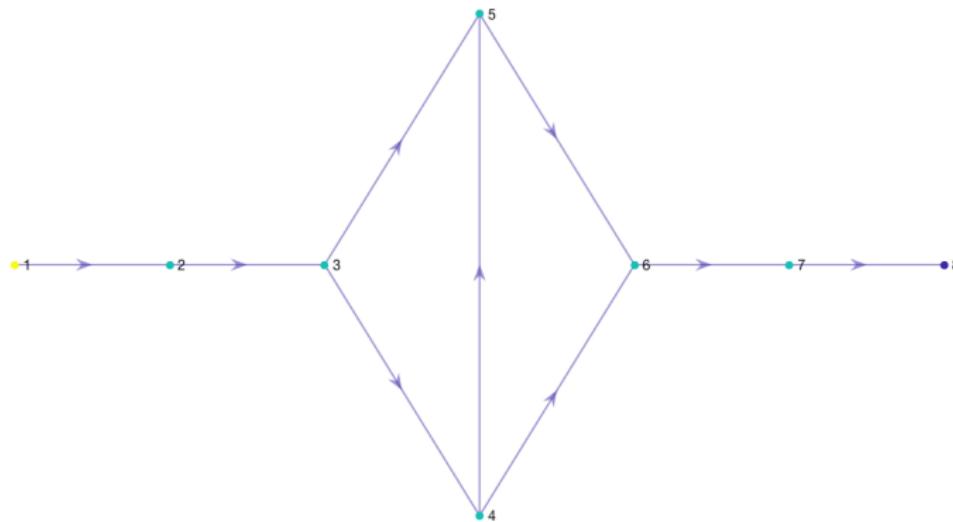


Figure: Topology of the diamond network



Numerical Simulation on the diamond network

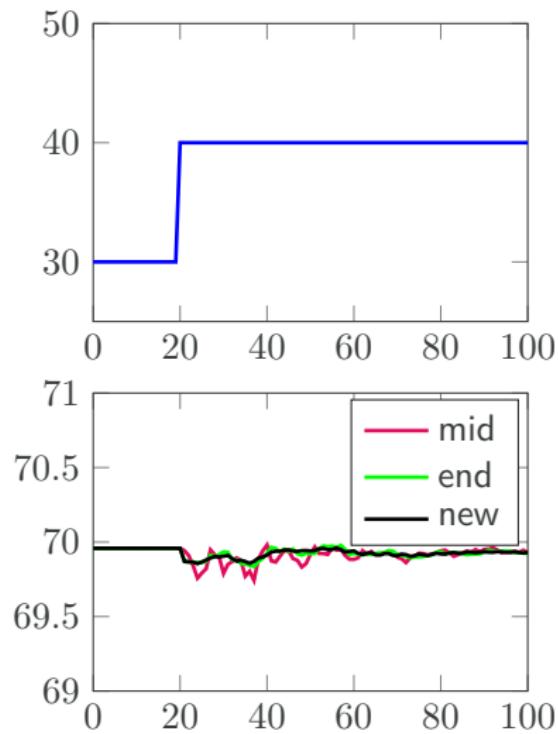
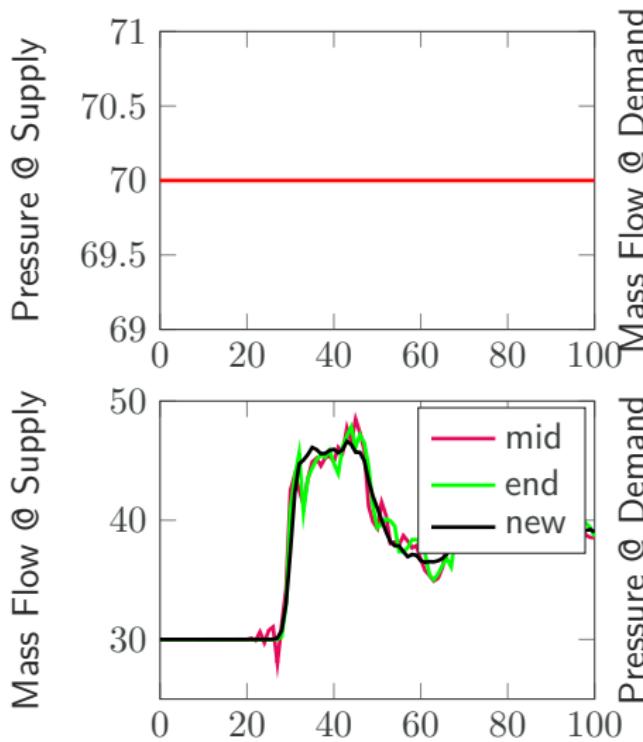




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Input-Output Systems

(Possibly nonlinear) Input-Output System:

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= g(x(t), u(t))\end{aligned}$$

- Input: $u : \mathbb{R} \rightarrow \mathbb{R}^M$
- State: $x : \mathbb{R} \rightarrow \mathbb{R}^N$
- Output: $y : \mathbb{R} \rightarrow \mathbb{R}^Q$
- Vector Field: $f : \mathbb{R}^N \times \mathbb{R}^M \rightarrow \mathbb{R}^N$
- Output Functional: $g : \mathbb{R}^N \times \mathbb{R}^M \rightarrow \mathbb{R}^Q$
- $N \gg 1, M \ll N, Q \ll N$

Input-to-Output Mapping:

$$u : \cdot \xrightarrow{\xi} : x : \cdot \xrightarrow{\eta} : y$$

- Is there a low(er) dimensional mapping $\eta \circ \xi : u \mapsto y$?
- Find transformation T such that $T(x)$ is sorted by I/O importance.
- System-theoretic approach: Quantify and balance ξ and η .



Empirical Gramians

Empirical Controllability and Observability Gramians [Lall et al'99]:

$$W_C = \int_0^{\infty} \xi(t) \xi^*(t) dt, \quad W_O = \int_0^{\infty} \eta^*(t) \eta(t) dt$$

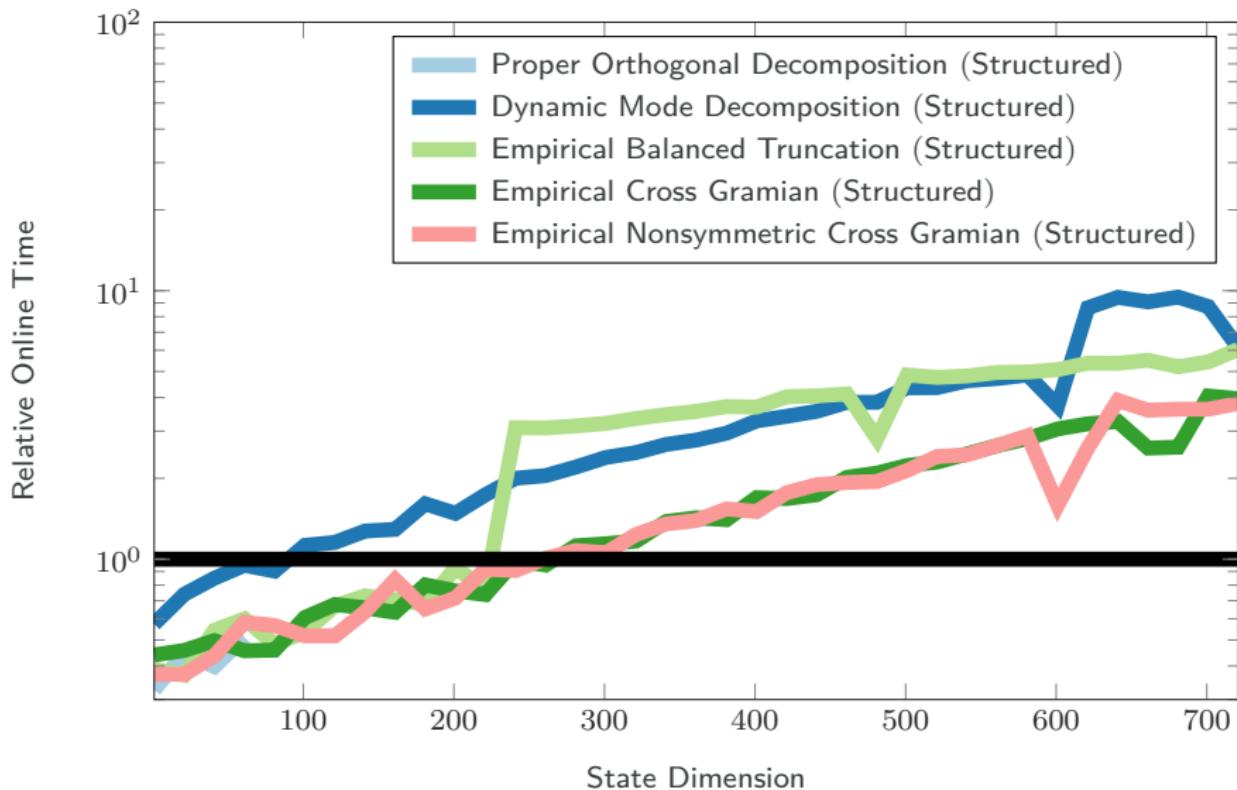
- Balancing W_C and W_O yields transformation T [Laub et al'87].
- Truncating T leads to reducing projection T_1 .
- Purely data-driven computation via ξ and η .
- Also applicable to unstable, parametric or implicit systems [Himpe'16].
- For linear (A,B,C) systems, this is *balanced truncation* [Moore'81].

Projection-Based Reduced Order Model:

$$\begin{aligned}\dot{x}_r(t) &= T_1 f(T_1^{-1} x_r(t), u(t)) \\ \tilde{y}(t) &= g(T_1^{-1} x_r(t), u(t))\end{aligned}$$

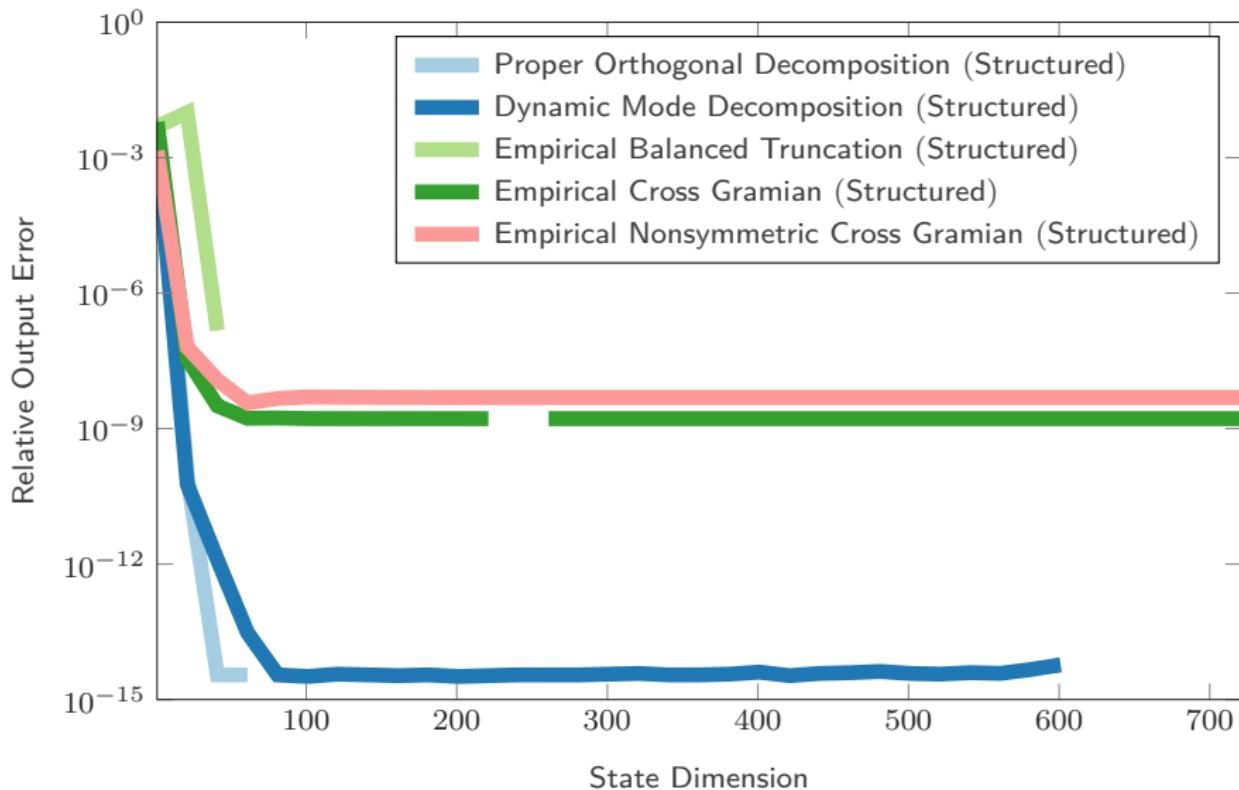


Model Order Reduction Online Time





Model Order Reduction L^∞ Error





Basic equation

$$\frac{\partial p}{\partial t} = -\frac{1}{\gamma z S} \frac{\partial q}{\partial x} \quad \frac{\partial q}{\partial t} = -S \frac{\partial p}{\partial x} - \frac{f_g \gamma z}{2DS} \frac{q|q|}{p}$$

Decoupled equation and solution

$$w^\pm = \frac{1}{2}(q \pm \sqrt{\gamma z} Sp) \quad \partial_t w^\pm \pm \frac{1}{\sqrt{\gamma z}} \partial_x w^\pm = \frac{1}{2} f(q, p)$$



Basic equation

$$\frac{\partial p}{\partial t} = -\frac{1}{\gamma z S} \frac{\partial q}{\partial x} \quad \frac{\partial q}{\partial t} = -S \frac{\partial p}{\partial x} - \frac{f_g \gamma z}{2DS} \frac{q|q|}{p}$$

Decoupled equation and solution

$$w^\pm = \frac{1}{2}(q \pm \sqrt{\gamma z} Sp) \quad \partial_t w^\pm \pm \frac{1}{\sqrt{\gamma z}} \partial_x w^\pm = \mathbf{0}$$

$$w^\pm(x, t) = w_0(x \mp \frac{1}{\sqrt{\gamma z}} t)$$



Hyperbolic Systems and Classical Model Order Reduction

Basic equation

$$\frac{\partial p}{\partial t} = -\frac{1}{\gamma z S} \frac{\partial q}{\partial x} \quad \frac{\partial q}{\partial t} = -S \frac{\partial p}{\partial x} - \frac{f_g \gamma z}{2DS} \frac{q|q|}{p}$$

Decoupled equation and solution

$$w^\pm = \frac{1}{2}(q \pm \sqrt{\gamma z} Sp) \quad \partial_t w^\pm \pm \frac{1}{\sqrt{\gamma z}} \partial_x w^\pm = 0$$

$$w^\pm(x, t) = w_0(x \mp \frac{1}{\sqrt{\gamma z}} t)$$

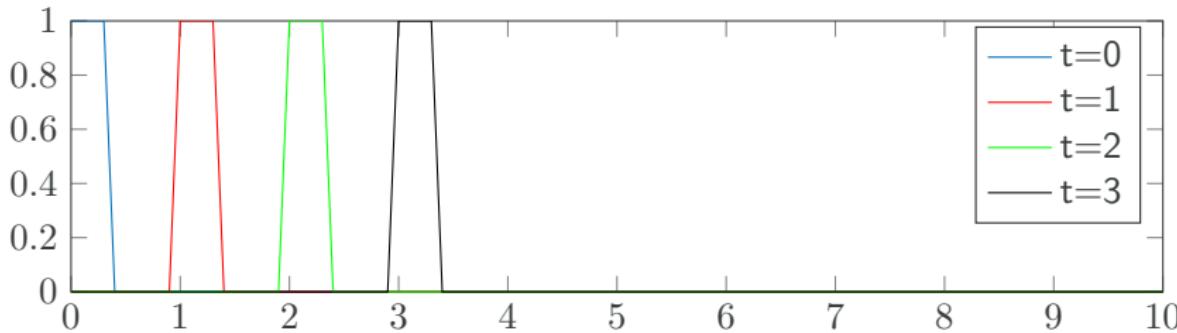




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Similar Work

- Rowley, Marsden, *Reconstruction equations and the karhunen-loeve expansion for systems with symmetry*, 2000
- Gerbeau, Lombardi, *Approximated lax pairs for the reduced order integration of nonlinear evolution equation*, 2014
- Peherstorfer, *Model reduction for transport-dominated problems via online adaptive basis and adaptive sampling*, 2018
- Nair, Balajewicz, *Transported snapshot model order reduction approach for parametric, steady-state fluid flows containing parameter-dependent shocks*, 2019
- Rim, Mandli, *Displacement interpolation using monotone rearrangement*, 2018
- Welper, *Interpolation of functions with parameter dependent jumps by transformed snapshots*, 2017
- ...



Hyperbolic Partial Differential Equation

The Model

$$\begin{aligned}\partial_t u(\cdot, \cdot, \mu) &= L(u(\cdot, \cdot, \mu), \mu) \text{ on } \Omega \times [0, T], \quad u(\cdot, 0, \mu) = u_0(\cdot, \mu) \text{ on } \Omega \\ \mathcal{G}(u(\cdot, \cdot, \mu), \mu) &= 0 \text{ on } \partial\Omega \times [0, T]\end{aligned}$$

- $\mu \in \mathcal{P} \subset \mathbb{R}$ with \mathcal{P} being a bounded parameter domain,
- T is the final time
- $u_0(\cdot, \mu)$ is the initial data
- $\Omega \subset \mathbb{R}^d$ is a bounded and open spatial domain
- $\mathcal{G}(\cdot, \mu)$ prescribes some boundary conditions
- $u(\cdot, t, \mu) \in \mathcal{X}$ and $u(x, t, \mu) \in \mathbb{R}$.
- $L(\cdot, \mu) : \mathbb{R} \rightarrow \mathbb{R}$ is of the form

$$L(\cdot, \mu) = -\nabla_x \cdot f(\cdot, \mu)$$



Solving the full order model

Finite dimensional approximation space

$$L^2(\Omega) \supset \mathcal{X}^N = \text{span}\{\phi_i : \phi_i = \frac{1}{\sqrt{\Delta x}} \mathbb{1}_{\mathcal{I}_i^x}, i \in \{1, \dots, N\}\}.$$

Above, $\mathbb{1}_A$ represents a characteristic function of the set $A \subset \mathbb{R}$, and $N = n_x$.

Using \mathcal{X}^N , we express the evolution equation for the FOM as

$$u^N(\cdot, t_{k+1}, \mu) = u^N(\cdot, t_k, \mu) + \Delta t \times L^N(u^N(\cdot, t_k, \mu), \mu), \quad \forall k \in \{1, \dots, K-1\},$$

where $L^N(\cdot, \mu) : \mathcal{X}^N \rightarrow \mathcal{X}^N$ is an approximation of the original $L(\cdot, \mu)$.



Core of the reduction problem

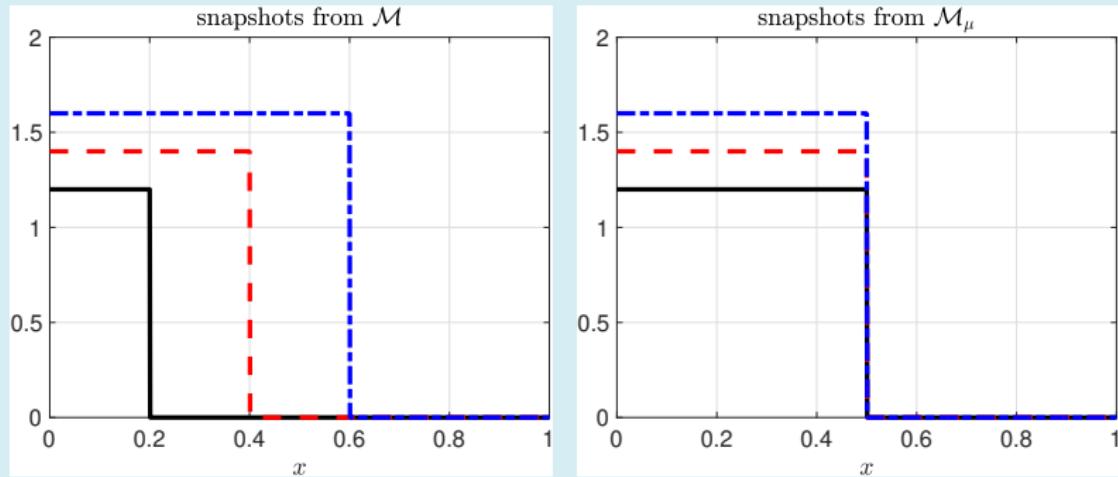


Figure: Snapshots taken from (a) \mathcal{M} and (b) \mathcal{M}_μ .



Core of the reduction problem

Consider the manifold $\mathcal{M} := \{f(\cdot, \mu) : \mu \in \mathcal{P}\} \subset L^2(\mathbb{R})$, where $f(\cdot, \mu)$ is a step function that scales and shifts to the right, and is given as

$$f(x, \mu) := \begin{cases} 1 + \mu, & x \leq \mu \\ 0, & x > \mu \end{cases}, \quad \mu \in \mathcal{P} := [0, 1].$$

Furthermore consider

$$\begin{aligned} \mathcal{M}_\mu &:= \{f(\varphi(\cdot, \mu, \hat{\mu}), \hat{\mu}) : \varphi(\cdot, \mu, \hat{\mu}) = x - (\mu - \hat{\mu}), \hat{\mu} \in \mathcal{P}\}, \\ &= \{\alpha f(\cdot, \mu) : \alpha \in [1, 2]\}. \end{aligned}$$



Core of the reduction problem

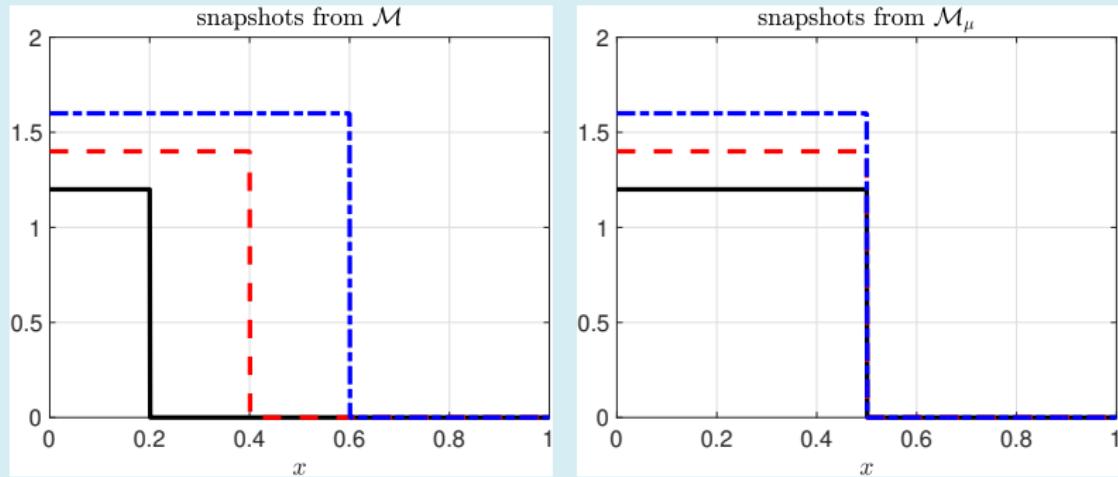


Figure: Snapshots taken from (a) \mathcal{M} and (b) \mathcal{M}_μ .



Reduced Solution manifold

For each time discretization point t_k we look for the solution in an approximation of this manifold;

$$\mathcal{M}_{\mu,t_k} := \{u^N(\varphi(\cdot, \mu, \hat{\mu}, t_k), t_k, \hat{\mu}) : \hat{\mu} \in \mathcal{P}\},$$

namely

$$u^N(\cdot, t_k, \mu) \approx u^n(\cdot, t_k, \mu) \in \mathcal{X}_{\mu,t_k}^n,$$

where $\mathcal{X}_{\mu,t_k}^n := \text{span}\{\psi_{\mu,t_k}^j : \psi_{\mu,t_k}^j = u^N(\varphi^M(\cdot, \mu, \hat{\mu}_j, t_k), t_k, \hat{\mu}_j), j \in \{1, \dots, M\}\}$.

We compute a solution in \mathcal{X}_{μ,t_k}^n using residual-minimisation. Writing the finite-volume scheme from above as a residual minimisation problem provides

$$u^N(\cdot, t_{k+1}, \mu) = \arg \min_{w \in \mathcal{X}^N} \| \text{Res}(w, u^N(\cdot, t_k, \mu)) \|_{\mathbb{R}^N}, \quad \forall k \in \{0, \dots, K-1\},$$

$$u^n(\cdot, t_{k+1}, \mu) = \arg \min_{w \in \mathcal{X}_{\mu,t_{k+1}}^n} \| \text{Res}(\Pi_{\mu,t_{k+1}} w, \Pi_{\mu,t_k} u^n(\cdot, t_k, \mu)) \|_{\mathbb{R}^N}.$$



Algorithms Summary

Offline Phase: Algorithm for model reduction

1. Compute the FOM for all $\mu \in \{\hat{\mu}_j\}_{j=1,\dots,M}$ using the time-evolution scheme and a finite volume scheme.
2. Compute all the snapshots of the spatial transforms $\{\varphi(x, \hat{\mu}_j, \hat{\mu}_l, t_k)\}_{j,l=1,\dots,M}$ for all $k \in \{1, \dots, K\}$.
3. Perform the offline phase of hyper-reduction.

Online Phase: Algorithm for model reduction

1. For a given μ , approximate $\{\varphi(x, \mu, \hat{\mu}_j, t_k)\}_{j=1,\dots,M}$ using polynomial interpolation.
2. Perform the online phase of hyper-reduction.
3. Compute $u^n(\cdot, t_k, \mu)$ for all $k \in \{1, \dots, K\}$ using residual-minimisation and hyper-reduction.



Numerical Experiments

Example

Two dimensional Burger's equation with parameterised initial data

$$\partial_t u(\cdot, \cdot, \mu) + \frac{1}{2} \partial_x u(\cdot, \cdot, \mu)^2 + \frac{1}{2} \partial_y u(\cdot, \cdot, \mu)^2 = 0, \text{ on } \Omega \times [0, T].$$

We choose $\mathcal{P} = [1, 3]$, $\Omega = [0, 1]$ and $T = 0.8$. The initial data is given as

$$u_0(x, \mu) = \begin{cases} \mu \times \exp\left(-1/\left(1 - \left(\frac{\|x - \delta_1\|}{\delta_2}\right)^2\right)\right), & \frac{\|x - \delta_1\|}{\delta_2} < 1 \\ 0, & \text{else} \end{cases}.$$

We set $\delta_1 = (0.5, 0.5)^T$ and $\delta_2 = 0.2$.

- N. Sarna, S.Grundel, **Model Reduction of Time-Dependent Hyperbolic Equations using Collocated Residual Minimisation and Shifted Snapshots** submitted



Quantitative Results

We compare S-ROM (snapshots based linear ROM) and SS-ROM (shifted snapshots based non-linear ROM).

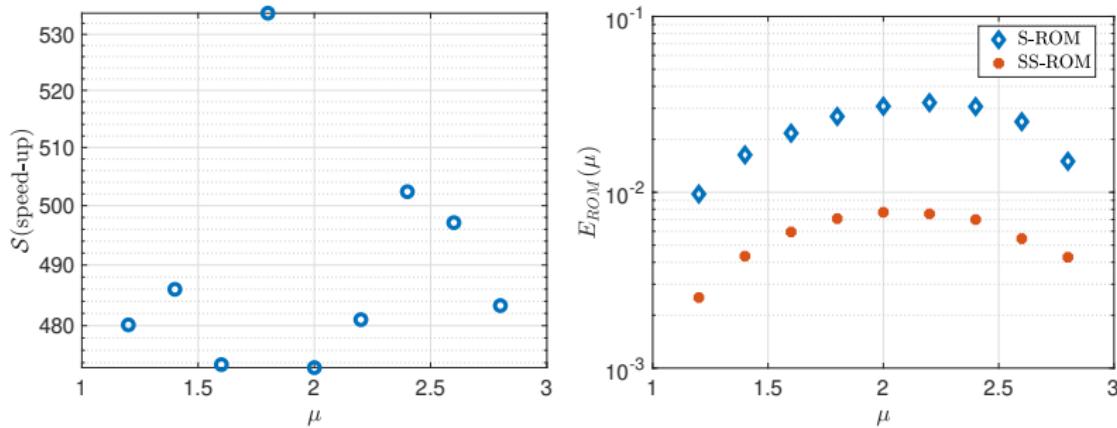


Figure: Results for test-2. (a) Speed-up resulting from SS-ROM; and (b) E_{ROM} resulting from S-ROM and SS-ROM. Fig-(b) has a y-axis on a log-scale.



Qualitative Results

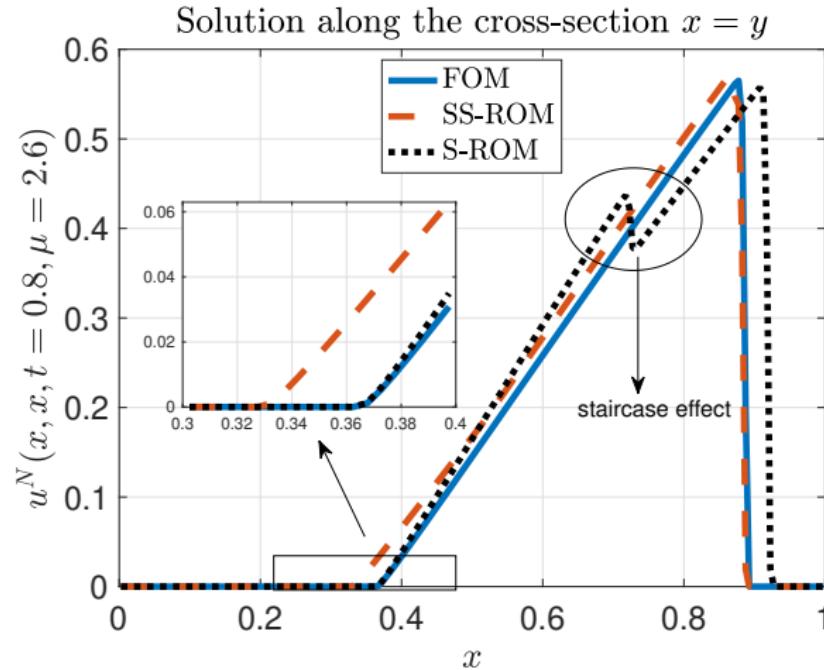


Figure: Results for test-2. FOM and the ROM along the cross-section $x = y$ for $\mu = 2.6$ and $t = 0.8$.



Two Feature Initial Condition

$$u_0(x) = \begin{cases} \mu \exp\left(-1/\left(1 - \left(\frac{x-\delta_1}{\delta_2}\right)^2\right)\right), & \left|\frac{x-\delta_1}{\delta_2}\right| < 1 \\ -\exp\left(-1/\left(1 - \left(\frac{x+\delta_1}{\delta_2}\right)^2\right)\right), & \left|\frac{x+\delta_1}{\delta_2}\right| < 1 \\ 0, & \text{else} \end{cases}$$



Limitations

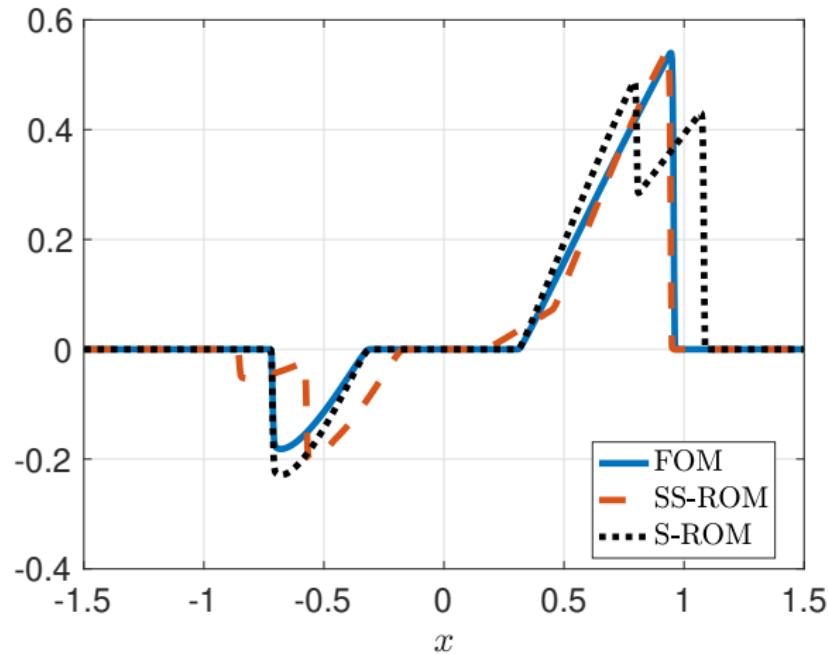


Figure: Results that show the limitation of SS-ROM. Computed with one-dimensional Burger's equation with the initial data as given in (36). The solutions are for $\mu = 2$ and $t = 1$.



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Preconditioning

Table: Computational time for the 1st Newton iteration

h	$\#D_F$	t_{S^1}	IDR(4)	backslash
40	1.03e+05	3.85	0.25	0.13
20	2.01e+05	8.12	0.52	0.36
10	3.97e+05	17.84	1.06	1.18
5	7.91e+05	38.44	2.13	1054.62
2.5	1.58e+06	81.42	4.34	-

FVM and Iterative Solvers²,

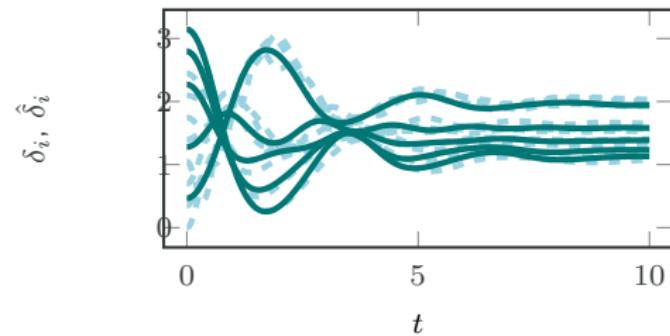
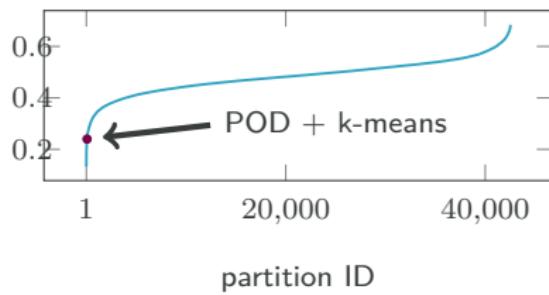
²Yue, Grundel, Stoll, Benner *Efficient Numerical Methods for Gas Network Modeling and Simulation*, arXiv



Dynamic Power Flow

Generator model (swing equation)

$$M_i \ddot{\delta}_i(t) + D_i \dot{\delta}_i(t) = P_i - \sum_{j=1}^N a_{ij} \sin(\delta_i(t) - \delta_j(t)),$$



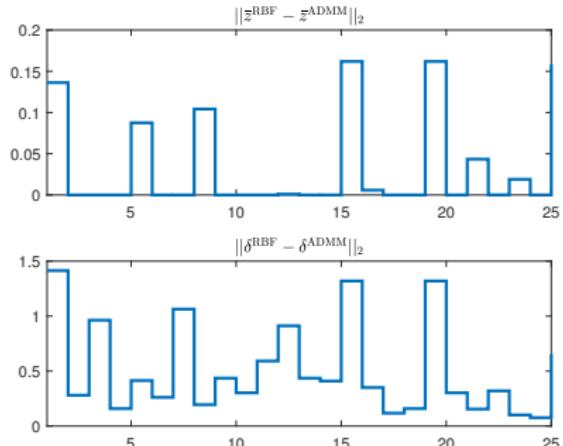
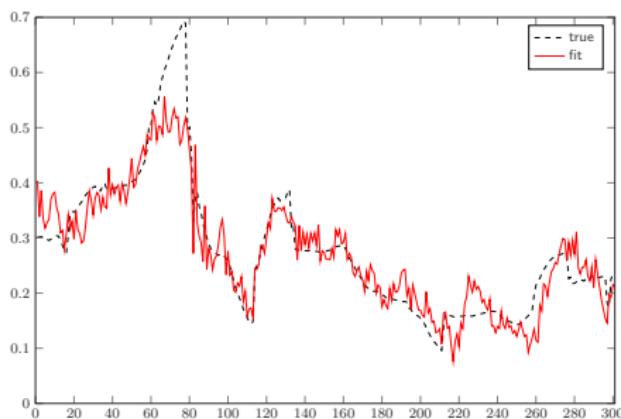
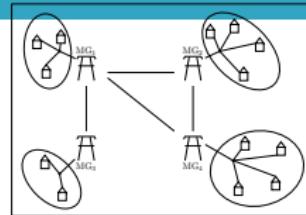
- Mlinarić, P., Grundel S., and Benner P. Decision and Control (CDC), 54th Annual Conference on. IEEE, 2015.
- Jongsma, H.-J.; Mlinarić, P.; Grundel, S.; Benner, P.; Trentelman, H. L.: Mathematics of Control, Signals, and Systems 30 (1), 6 (2018)



Coupled Microgrids

We consider four microgrids (MGs), connected via transportation lines.

- Grundel S., Sauerteig P., Worthmann K.,
Surrogate models for coupled microgrids
Progress in Industrial Mathematics at
ECMI 2018,



Error in local approximation 10^{-2} but in global problem 10^{-1}



Bidirectional Optimization of coupled microgrids

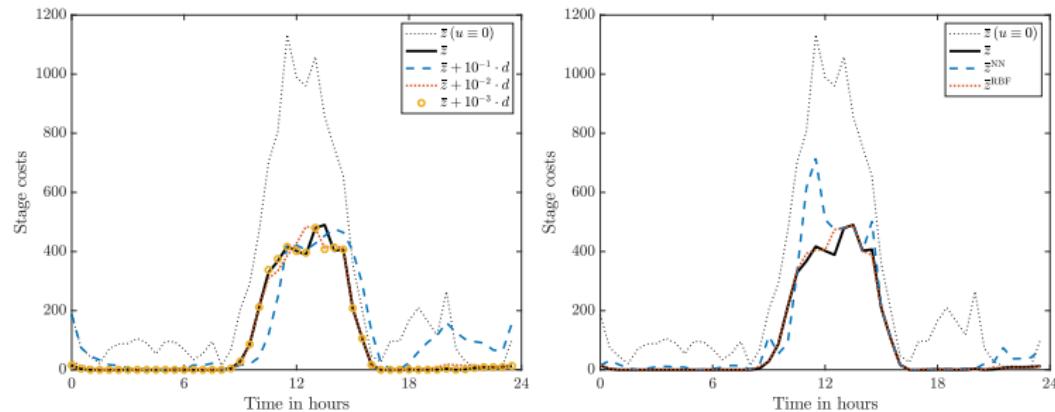


Figure: Impact of mapping error (top) and approximation via RBF and NN (bottom) on the costs within 48 consecutive time steps.

- M. Baumann, S. Grundel, P. Sauerteig, and K. Worthmann. **Surrogate models in bidirectional optimization of coupled microgrids.** *at-Automatisierungstechnik*, 67(12), 1035-1046, 2019.



Thank you for your attention

